Probing the curvature of the Universe from supernova measurement

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Abstract

We study the possibility to probe the spatial geometry of the Universe by supernova measurement. We illustrate with an accelerating universe model with infinite-volume extra dimensions, for which the 1σ level supernova results indicate that the Universe is closed.

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The precision measurements of the Wilkinson Microwave Anisotropy Probe (WMAP) have provided high resolution Cosmic Microwave Background (CMB) data [1, 2] and elevated cosmology to a new maturity. Among interesting conclusions that have been reached from these data, the WMAP results indicate that while flatness of the Universe is confirmed to a spectacular precision on all but the largest scales [1], a closed universe with positively curved space is marginally preferred [3, 4, 5, 6, 7]. This tendency of preferring closed universe is not restricted to the WMAP data, it appeared in a suite of CMB experiments before [8, 9, 10]. The improved precision from WMAP provides further confidence.

In addition to CMB, recently it was argued that the cubic correction to the Hubble law measured with high-redshift supernovae is another cosmological measurement that probes directly the spatial curvature [11]. This is the first non-CMB probe of the spatial geometry, which can provide a cross-check to the result got by CMB. In a toy model, it was already found that a curvature radius is larger than the Hubble distance [11].

Our Universe is accelerating rather than decelerating. This may be regarded as the evidence for a nonzero but very small cosmological constant (see [12] for a review and related references in [13]). Another possibility is that the phenomenon of accelerated expansion is caused by a breakdown of the standard Friedmann equation due to the extra-dimensional contribution [14, 15, 16, 17]. Studies on this possibility can also be found in [18]. In this work we will consider the accelerated universe model resulted from the gravitational leakage into extra dimensions [16]. We will attempt to extract information from the full redshift data to test the spatial geometry.

Consider the accelerating universe described by the model with infinite-volume extra dimensions [16], the Friedmann equation is expressed as

$$H^2 + \frac{k}{a^2} = \left\{ \sqrt{\frac{\rho}{3M_p^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right\}^2 , \tag{1}$$

where ρ is the total cosmic fluid energy density and r_c is the crossover scale. Eq. (1) can also be recasted in terms of the redshift as

$$H^2 = H_0^2 \left\{ -\Omega_k(1+z)^2 + \left[\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_M(1+z)^3} \right]^2 \right\} , \tag{2}$$

where $\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}$, Ω_M is the non-relativistic matter density. The conservation for energy-momentum tensor of the cosmic fluid is still described by

$$\dot{\rho} + 3H(\rho + P) = 0. \tag{3}$$

Using definitions $q_0 = -\frac{\ddot{a}a}{\dot{a}^2}|_0$, $j_0 = \frac{\dddot{a}a^2}{\dot{a}^3}|_0$ with dot denoting the differentiation with respect to time t for the deceleration parameter and the "jerk", respectively, we have directly from equation (2)

$$q_0 = -(1+\Omega_{k_0}) + \frac{3\Omega_M\sqrt{1+\Omega_{k_0}}}{2\sqrt{\Omega_{r_c}+\Omega_M}}, \tag{4}$$

$$j_0 = (1+\Omega_{k_0}) - \frac{9\Omega_M^2\sqrt{\Omega_{r_c}}}{4(\Omega_{r_c}+\Omega_M)^{3/2}}, \tag{5}$$

where the normalization of (2) at the present epoch

$$\Omega_{r_c} = \frac{(1+\Omega_{k_0}-\Omega_M)^2}{4(1+\Omega_{k_0})} \tag{6}$$

has been employed. With (6), q_0 and j_0 are only determined by Ω_M and Ω_{k_0}.

The physically reasonable cosmic model has the following requirements [19]: (1) the total density is currently not increasing as a function of time; (2) for causality and stability, the present sound speed c_s of the total system satisfies $0 \leq c_s^2 \leq 1$.

Employing (1) and (3), the variation of the total cosmic fluid energy density and the sound speed of the total cosmic fluid at the present epoch are

$$\dot{\rho}|_0 = -6M_p^2H_0^3(1+q_0+\Omega_{k_0})\left[1-\frac{\sqrt{\Omega_{r_c}}}{\sqrt{1+\Omega_{k_0}}}\right], \tag{7}$$

$$\begin{aligned} c_s^2 &= \frac{\dot{P}}{\dot{\rho}}|_0 \\ &= \frac{(j_0-1-\Omega_{k_0})(1-\sqrt{\Omega_{r_c}}/\sqrt{1+\Omega_{k_0}}) + (q_0+1+\Omega_{k_0})^2\sqrt{\Omega_{r_c}}/(1+\Omega_{k_0})^{3/2}}{3(1+q_0+\Omega_{k_0})[1-\sqrt{\Omega_{r_c}}/\sqrt{1+\Omega_{k_0}}]}. \end{aligned} \tag{8}$$

The first requirement implies

$$(1+q_0+\Omega_{k_0})(1-\frac{\sqrt{\Omega_{r_c}}}{\sqrt{1+\Omega_{k_0}}}) \geq 0. \tag{9}$$

Employing (6) and the fact that $|\Omega_{k0}| \leq 0.1$ as a consequence of CMB data, the above requirement reduces to

$$1+q_0+\Omega_{k_0} \geq 0. \tag{10}$$

Using (4), we see that Eq. (10) can obviously be satisfied.

The second requirement can now be written in a simplified form as

$$f_1 \leq j_0 \leq f_2, \tag{11}$$

where $f_1 = (1+\Omega_{k_0}) - \frac{(1+\Omega_{k_0}-\Omega_M)(q_0+1+\Omega_{k_0})^2}{(1+\Omega_{k_0})(1+\Omega_{k_0}+\Omega_M)}$ and $f_2 = 4(1+\Omega_M)+3q_0 - \frac{(1+\Omega_{k_0}-\Omega_M)(q_0+1+\Omega_{k_0})^2}{(1+\Omega_{k_0})(1+\Omega_{k_0}+\Omega_M)}$. Substituting Eqs. (4) and (6) into the expression of f_1, we find that $j_0 = f_1$, which means that the sound speed of the total system in this model is exactly zero.

We now turn to determine the cosmological density parameters from the supernova (SN) Ia data compiled by Riess et al. [20]. The likelihood for the parameters Ω_M and Ω_{k_0} can be obtained from a χ^2 statistics [20, 21], where

$$\chi^2(H_0, \Omega_M, \Omega_{k_0}) = \sum_i \frac{[\mu_{p,i}(z_i, H_0, \Omega_{k_0}, \Omega_M) - \mu_{o,i}]^2}{\sigma_i^2}, \tag{12}$$

$\mu_p = 5\log_{10}(d_L/\mathrm{Mpc}) + 25$ and μ_o are distance modulus for the model and the observations, respectively. d_L is the luminosity distance defined for the Friedmann-Robertson-Walker universe model as

$$\begin{aligned} d_L &= a_0(1+z)r_1 \qquad (13) \\ &= \begin{cases} a_0(1+z)\sin[\frac{1}{a_0 H_0}\int_0^z \frac{dz'}{E(z')}], & \text{closed} \\ \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')}, & \text{flat} \\ a_0(1+z)\sinh[\frac{1}{a_0 H_0}\int_0^z \frac{dz'}{E(z')}], & \text{open} \end{cases} \end{aligned}$$

for closed, flat and open universes respectively. The function $E(z)$ quantifies the expansion rate as a function of redshift defined as $H(z) = H_0 E(z)$. σ_i in (12) is the total uncertainty in the observation. Marginalizing our likelihood function over the nuisance parameter H_0 by integrating the likelihood function $L = \exp(-\chi^2/2)$ over all possible values of H_0 with a flat prior assumption on H_0, yields the confidence intervals shown in Fig. 1 by combining Eqs. (2), (6), (12) and (13).

Using the contour Ω_{k_0}, Ω_M values, we can get the corresponding q_0, j_0 and f_1 as plotted in Fig. 2. Note that the contours shown here are from the gold sample SN Ia data compiled in [20].

Lines added in Fig.2 show the second requirement for a reasonable cosmic model, $j_0 = f_1$, with Ω_M varying in the range [0.2-0.4] and different Ω_{k_0} for open, flat and closed universes, respectively. It is clear that in the 2σ level, there are only overlaps with the supernova data for $\Omega_{k_0} > 0$. This corresponds to say that the data favors the closed universe almost at 2σ level.

To obtain tighter constraints on the parameter space, we also include constrains from

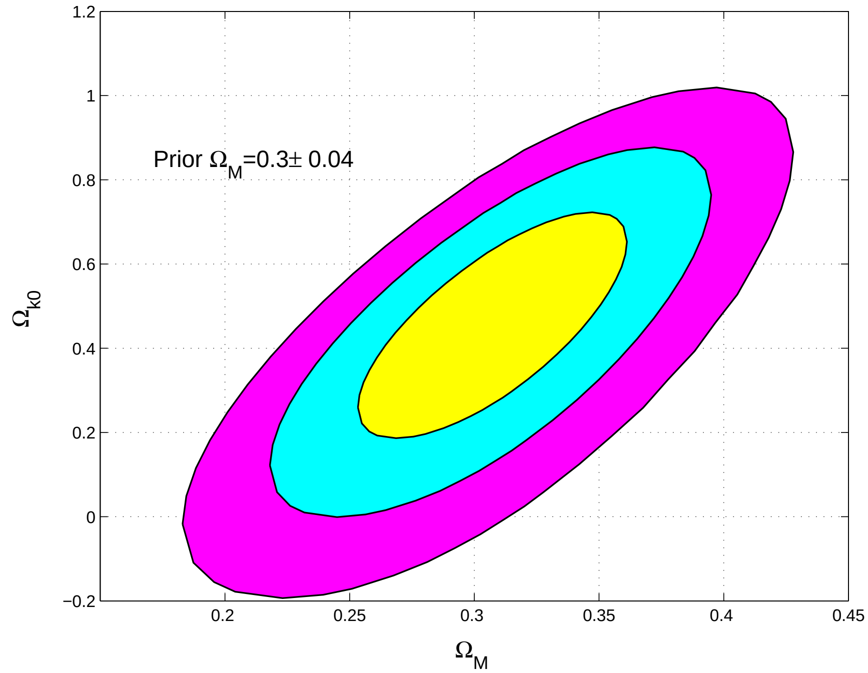

Figure 1: The 1σ, 2σ and 3σ confidence contours for Ω_M and Ω_{k0} with the prior $\Omega_M = 0.3 \pm 0.04$ [23].

combined WMAP data [1, 2] and SN Ia data. We minimize

$$\chi^2 = \sum_i \frac{[\mu_{p,i}(z_i, H_0, \Omega_{k0}, \Omega_M) - \mu_{o,i}(z_i)]^2}{\sigma_i^2} + \frac{[\mathcal{R}_p(\Omega_{k0}, \Omega_M) - \mathcal{R}_o]^2}{\sigma_{\mathcal{R}}^2}, \tag{14}$$

where $\sigma_{\mathcal{R}}$ is the uncertainty in $\mathcal{R}$, the CMB shift parameter $\mathcal{R} \equiv \Omega_M^{1/2} H_0 r_1(z_{\rm ls}) = 1.710 \pm 0.137$ [22] and $z_{\rm ls} = 1089 \pm 1$ [1, 2]. The results are shown in Fig.3. The combined constraints give $\Omega_M = 0.25^{+0.05}_{-0.04}$ and $\Omega_{k0} = 0.01^{+0.09}_{-0.08}$. This shows that in the absence of positive spatial curvature, Ω_M tends to take a smaller value. It implies that from the observed Ω_M around 0.3, we should have the positive curvature.

From the SN Ia data, Ω_{r_c} is constrained to be 0.23 ($[0.18, 0.28]$ in 1σ region; $[0.14, 0.31]$ in 2σ region and $[0.1, 0.33]$ in 3σ region); combined with CMB, we have tighter constraint, $\Omega_{r_c} = 0.14$ ($[0.12, 0.16]$ in 1σ region; $[0.11, 0.17]$ in 2σ region and $[0.10, 0.18]$ in 3σ region). The corresponding crossover scale $r_c = 1.04 H_0^{-1}$ from supernova data and $r_c = 1.34 H_0^{-1}$ from combined CMB and SN Ia data. This constrained parameter is in good agreement with the result comes from lunar laser ranging experiments that monitor the moon's perihelion procession with a great accuracy [24].

In summary, we have probed the geometry of a specific model describing the accelerating

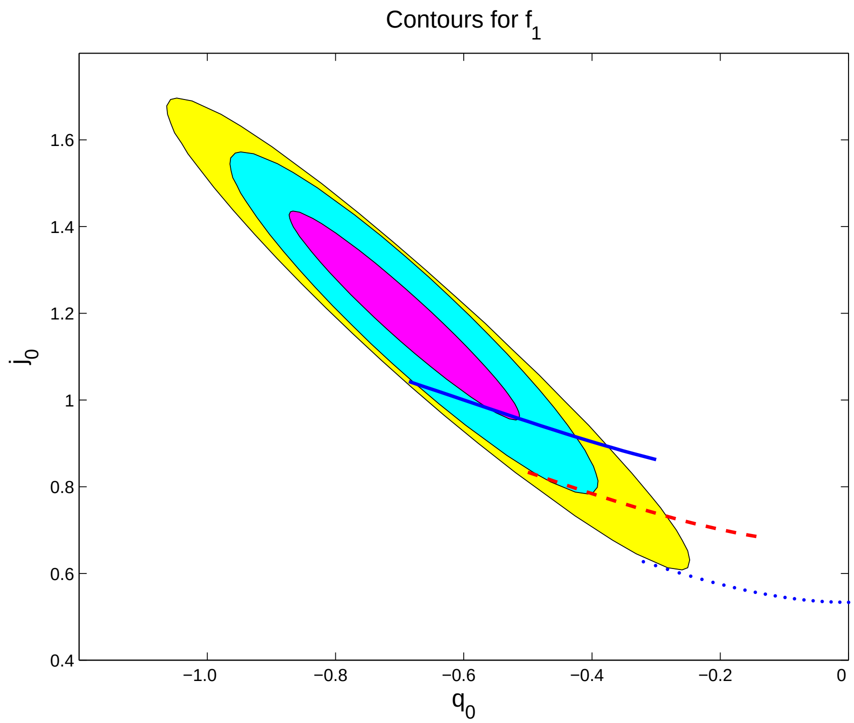

Figure 2: The 1σ, 2σ and 3σ confidence contours for q_0 and j_0. The solid, dashed and dotted lines are plots for $j_0 = f_1$ with Ω_M varying in the range [0.2-0.4] and $\Omega_{k_0} = 0.2, 0, -0.2$ respectively.

universe by using the full redshift data in supernova measurements. To almost 2σ level, our result indicates that the universe is closed. This result is also favored by including WMAP data constraint, which agrees to a suite of CMB experiments. The result obtained is consistent with the interpretation from other models, e.g. the matter plus cosmological constant case, that the Riess et al. data show a tendency towards a closed universe. Of course it is too early to draw conclusions just on 2σ level data, and we expect that future supernova measurements can determine the spatial curvature precisely.

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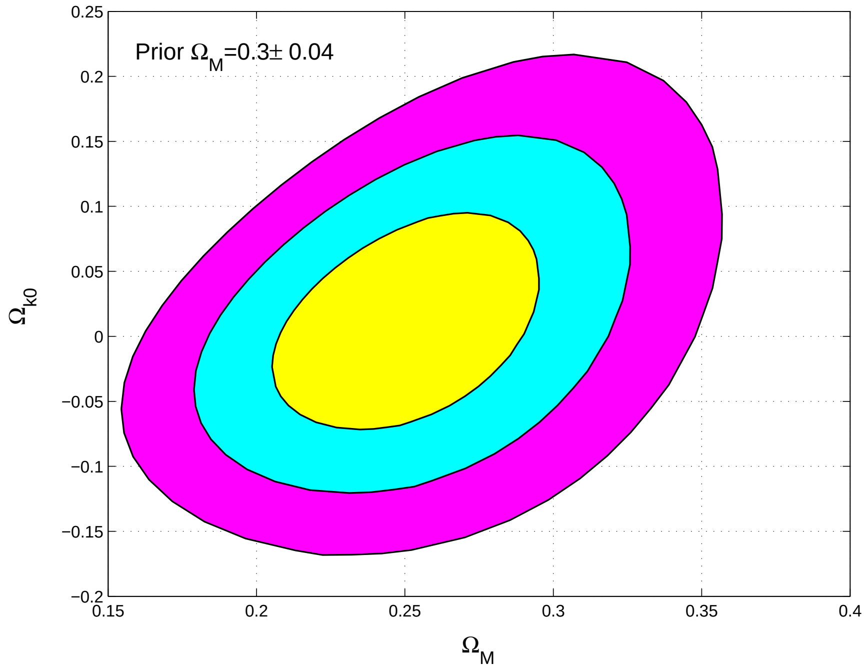

Figure 3: The 1σ, 2σ and 3σ confidence contours for Ω_M and Ω_{k0} by combining the CMB data and supernova data with the prior $\Omega_M = 0.3 \pm 0.04$.

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[1] C. L. Bennett et al., Astrophys.J.Suppl. 148 (2003) 1.

[2] D. N. Spergel et al., Astrophys.J.Suppl. 148 (2003) 175.

[3] J. Uzan, U. Kirchner and G. F. R. Ellis, Mon.Not.Roy.Astron.Soc. 344 (2003) L65.

[4] G. F. R. Ellis and R. Maartens, Class.Quant.Grav. 21 (2004) 223.

[5] A. Linde, JCAP 0305 (2003) 002

[6] G. Efstathiou, Mon.Not.Roy.Astron.Soc. 343 (2003) L95; M. Tegmark, A. de Oliveira-Costa and A. Hamilton, Phys.Rev. D68 (2003) 123523.

[7] J. P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J. Uzan, Nature 425 (2003) 593.

[8] J. L. Sievers et al., Astrophys.J. 591 (2003) 599-622.

[9] C. B. Netterfield et al., Astrophys. J. 571 (2002) 604.

[10] A. Benoit et al., Astron.Astrophys. 399 (2003) L25-L30; ibid 399 (2003) L19-L23.

[11] R. R. Caldwell and M. Kamionkowski, astro-ph/0403003.

[12] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75 (2003) 559.

[13] A. G. Riess et al., Astron.J. 116 (1998) 1009; M. Li, hep-th/0403127; Q. G. Huang and Y. Gong, JCAP 0408 (2004) 006; S. Hsu, A. Zee, hep-th/0406142.

[14] G. Dvali, G. Gabadaze and M. Porrati, Phys. Lett. B 485 (2000) 208.

[15] G. Dvali and G. Gabadaze, Phys. Rev. D 63 (2001) 065007.

[16] C. Deffayet, G. Dvali and G. Gabadaze, Phys. Rev. D 65 (2002) 044023.

[17] G. Dvali and M. S. Turner, astro-ph/0301510.

[18] E. V. Linder, Phys. Rev. D 70 (2004) 023511; B. Wang, E. Abdalla and R. K. Su, hep-th/0404057; Y. Gong and C.K. Duan, Class. Quant. Grav. 21 (2004) 3655; Mon. Not. Roy. Astron. Soc. 352 (2004) 847; Y. Gong, X.M. Chen and C.K. Duan, Mod. Phys. Lett. A 19 (2004) 1933.

[19] T. Chiba, T. Nakamura, Prog.Theor.Phys. 100 (1998) 1077.

[20] A.G. Riess et al., Astrophys. J. 607 (2004) 665.

[21] B. J. Barris et al., Astrophys.J. 602 (2004) 571-594.

[22] J.R. Bond, G. Efstathiou and M. Tegmark, Mon. Not. Roy. Astron. Soc. 291 (1997) L33; A. Melchiorri, L. Mersini, C.J. Ödman and M. Trodden, Phys. Rev. D 68 (2003) 043509; Y. Wang and P. Mukherjee, Astrophys. J. 606 (2004) 654; U. Alam, V. Sahni and A.A. Starobinsky, JCAP 0406 (2004) 008.

[23] M. Tegmark et al., Phys. Rev. D 69 (2004) 103501.

[24] A. Lue, G. Starkman, Phys. Rev. D 67 (2003) 064002; G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D 68 (2003) 024012.